TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 575

INVESTIGATION OF THE VARIATIONS IN THE VELOCITY OF THE AIR FLOW ABOUT A WING PROFILE

By Walter Repenthin

From Zeitschrift für Flugtechnik und Motorluftschiffahrt
July 15, 1929

Washington
July, 1930

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL MEMORANDUM NO. 575.

INVESTIGATION OF THE VARIATIONS IN THE VELOCITY OF THE AIR FLOW ABOUT A WING PROFILE.*

By Walter Repenthin.

The mounting of either a tractor or pusher propeller near a streamlined wing not only affects the efficiency of the propeller, but also produces variations in the flow which, being always unsymmetrical, may subject the propeller to additional aerodynamic stresses. The purpose of the present investigation was therefore to determine the variations in the velocity of the air when it was obliged to flow about a wing profile. Of especial interest was the determination of the velocity as compared with that of the undisturbed flow and of the velocity drop in successive planes perpendicular to the flow direction at increasing distances from the wing profile. On the basis of these results, it is then possible to obtain an approximate picture of the asymmetry of the loading of the propeller disk at different positions of the propeller with respect to the wing profile.

In order to enable a theoretical calculation of the flow relations about a profile, we must choose profiles susceptible of hydrodynamic and mathematical investigation, like the Joukows-

*"Untersuchungen über die Veränderung der Windgeschwindigkeit bei Umströmung eines Flugelprofiles." From Zeitschrift für Flugtechnik und Motorluftschiffahrt, July 15, 1929, pp. 341-347.

ky profiles. The possible variants in the construction of such a profile were so correlated that the resulting chord, thickness and camber approximately agreed with the corresponding dimensions of the profiles in actual use.

By taking such a profile as the basis, it is possible, with the aid of mathematical formulas employed in conformal transformation for the development of wing profiles from a circle, to transfer the relatively easily determined flow relations with respect to the course and the velocity to the corresponding points of the field of flow about the J-profile, thus rendering it possible to determine the theoretical flow relations in the vicinity of the profile to be investigated. It is not possible by theoretical investigation to determine the effect of the boundary zone in the immediate vicinity of the profile, where laminar flow conditions prevail which are not directly susceptible of theoretical calculation. The boundary zone in question is so small, however, that it can be disregarded in practice.

The selected J-profile is the Göttingen profile No. 433. It has the characteristic parameter for the camber, namely, the approximate ratio of the rise of a circular arc, plotted as a curved profile axis, to the corresponding half-chord $f/l = 0.1$. For the thickness, the approximate ratio of the maximum profile thickness to the profile chord $d/l = 0.15$. This profile was constructed from these values, as shown in Figure 1.

The angle of attack then had to be determined for this profile, since the effect of the flow velocity depends on this angle. It appeared to be the most important to investigate the flow relations at sea level, for which an angle of attack of 0°, or somewhat less, has been found to be the best. The adoption of this angle of attack gives the same angle of attack for the generating fundamental circle and establishes the position of the points of full dynamic pressure on which the calculation of the circulation and the course of the unsymmetrical flow about the circular profile is based. The details of this method are fully explained in the later writings of E. Trefftz and Oskar Schrenk. Hence only a brief summary will be given here. The unsymmetrical flow about the circular profile, which corresponds to the determining flow for the wing profile, may be considered as the superposing of a symmetrical flow about the circular profile and a circulatory flow about the same. The general potential equation of the two-dimensional flow about a circular profile is written as follows:

For the symmetrical flow,

$$f(Z) = - V_\infty' \left(Z + \frac{R_1^2}{Z}\right) \tag{1}$$

For the circulatory flow,

$$f(Z) = - i \frac{\Gamma}{2\pi} \ln Z \tag{2}$$

Z represents the complex distance of any point in the Z-plane from the origin of the coordinates and is therefore

The introduction of complex coordinates is necessitated by the principle of conformal or orthomorphic transformation.

V_∞ represents the flow velocity outside the disturbed region around the cylinder.

R_1 represents the radius of the fundamental or basic circle ($R_1 = X_1^2 + Y_1^2$).

Γ represents the circulation corresponding to the location of the center of dynamic pressure on the fundamental circle.

With the introduction of the coordinates $X + iY$ in the above functional equations and the division into real and imaginary parts, the former represents the potential and the latter the stream function. We obtain for them, with a few transformations in the second case, the following equations:

$$\psi(X,Y) = V_\infty \left(Y - \frac{R_1^2\, Y}{X^2 + Y^2}\right) \tag{3}$$

$$\psi(X,Y) = -\frac{\Gamma}{2\pi} \ln \sqrt{X^2 + Y^2} \tag{4}$$

By putting each of the two functional equations equal to a constant C_1 or C_2, we can determine the course of a streamline, if we solve the equations for X and Y and calculate, for the assumed value of one of the unknowns, the corresponding value of the other unknown. Variations in the value of C thus enable the determination of a whole group of streamlines. Equation (4) requires only a knowledge of the magnitude of Γ, or Γ can be

replaced by an expression containing only quantities which can be measured from the drawing. This result is attained by the following line of reasoning.

By differentiating the potential functions, we obtain the velocity functions, which give for equations (1) and (2), after suitable transformations, the following expressions for the absolute velocity:

$$|W_p| = |2\ V_\infty \sin \Lambda| \quad \text{for the parallel flow} \tag{5}$$

$$|W_s| = \frac{\Gamma}{2\pi} \frac{1}{\sqrt{X^2 + Y^2}} \quad \text{for the circulatory flow} \tag{6}$$

The angle Λ is here formed by a line parallel to the direction of flow through the center of the fundamental circle and a line from the same point to the corresponding point of the field of flow with the coordinates X and Y. Equation (5) is a limiting form and is fulfilled for all points of the fundamental circle. It also holds good for the centers of dynamic pressure on the same circle. At this point, however, the velocity of the parallel flow must equal the velocity of the circulatory flow, since the velocity is zero at the points of full dynamic pressure. For these points we therefore have

$$2\ V_\infty \sin \Lambda = \frac{\Gamma}{2\pi\sqrt{X_1^2 + Y_1^2}} = \frac{\Gamma}{2\pi R_1} \tag{7}$$

For this point the angle Λ has a definite value α_{th} which can be measured on the drawing. Hence we have

$$\Gamma = 4 \pi R_1 V_\infty \sin \alpha_{th} \tag{8}$$

or

$$\psi(X,Y) = 2 R_1 V_\infty \sin \alpha_{th} \ln \sqrt{X^2 + Y_2} \tag{9}$$

Both groups of curves for the circulatory flow and for the parallel flow can now be calculated and plotted by transforming the equations (3) and (4) to the form

$$X = \sqrt{\frac{Y^3 - C_1 Y^2 - Y R_1^2}{C_1 - Y}} \tag{10}$$

$$X = \sqrt{e^{\frac{2 C_2}{2 R_1 \sin \alpha_{th}}} - Y^2} \tag{11}$$

The last equation has the form of the general equation for a circle. Hence, on putting the term Y^2 equal to zero, X gives the radius:

$$X = e^{\frac{C_2}{2 R_1 \sin \alpha_{th}}} \tag{12}$$

The curves in Figure 2 are obtained from equations (10) and (12). Their calculated values are given in Tables I and II. Attention is called to the fact that, for the numerical calculation, R_1 is taken as the unit of mass and accordingly equals 1, and the values of X and Y are therefore multiples of the cylinder radius.

The two sets of curves plotted from these numerical values correspond therefore to the parallel flow and to the circulatory flow, but not to the unsymmetrical flow, which must satisfy the

functional equation

$$\psi(X,Y) = V_\infty \left(Y - \frac{R_1^2 \ Y}{X^2 + Y^2} - 2\ R_1 \sin \alpha_{th} \ln \sqrt{X^2 + Y^2}\right) \quad (13)$$

The numerical solution of this equation is extremely difficult, so that it is possible to calculate only individual points of irregularly staggered flow constants by the introduction of any convenient X and Y values. The streamlines can be easily determined, however, by the graphic combination of both members of the equation. Hence the above-described method was adopted at the outset. Figure 2 also shows the new set of curves for the unsymmetrical flow, as derived from the two original sets.

TABLE I

$$X = \sqrt{\frac{Y^3 - C\ Y^2 - Y\ R_1}{C - Y}};\quad C < Y,\ R_1 = 1$$

$C_1 = 0$	Y	0 1	Circumference of Circle					
	X	1 0						
$C_1 = 0.1$	Y	1.05	0.75	0.5	0.25	0.15	0.14	0.1
	X	0	0.768	1.0	1.305	1.725	1.86	∞
$C_1 = 0.25$	Y	1.133	1.0	0.75	0.5	0.3	0.25	
	X	0	0.576	0.969	1.32	2.43	∞	
$C_1 = 0.4$	Y	1.22	1.0	0.75	0.5	0.4		
	X	0	0.815	1.26	2.18	∞		
$C_1 = 0.6$	Y	1.34	1.0	0.75	0.65	0.6		
	X	0	1.225	2.04	3.54	∞		
$C_1 = 0.8$	Y	1.478	1.25	1.0	0.85	0.8		
	X	0	1.105	2.0	4.04	∞		
$C_1 = 1.0$	Y	1.62	1.50	1.25	1.0			
	X	0	0.872	1.85	∞			
$C_1 = 1.2$	Y	1.77	1.5	1.2				
	X	0	1.66	∞				
$C_1 = 1.4$	Y	1.92	1.75	1.6	1.4			
	X	0	1.41	2.34	∞			
$C_1 = 1.6$	Y	2.08	2.0	1.6				
	X	0	1.0	∞				
$C_1 = 0.05$	Y	0.125						
	X	1.284						

TABLE II

$$X = e^{\frac{C_2}{2\,R_1\,\sin\,\alpha_{th}}};\quad \sin\,\alpha_{th} = 0.1582;\quad R_1 = 1$$

$$\log\,X = C_2\,\frac{\log\,e}{2\,\sin\,\alpha_{th}};\quad \log\,X = C_2\;1.371$$

C_2	0	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5	0.55
Y	0	0	0	0	0	0	0	0	0	0
X	1	1.171	1.371	1.605	1.881	2.201	2.580	3.536	4.849	5.675

Moreover, the course of the flow around the basic circular profile is well known. According to the principle of conformal transformation there corresponds to every point in the Z plane outside the basic circle a definite point within the auxiliary circle of the profile construction, whose location is determined by the following conditions.

1. To the line from the point 0 of the profile construction there corresponds a line mirrored with respect to the horizontal and passing through the zero point, which serves as a common axis of reference.

2. The length of this line is l^2/Z, where Z denotes the length of the upper line.

The bisection of the line between these two points gives the locatiom of the point of the z-plane corresponding to the outer point of the Z-plane for which the profile is also constructed. Strictly speaking, we obtain by this profile construction all the measurements in the z-plane only half-scale, but

this is of no consequence in the present velocity calculation, since it is corrected by using the coefficient 2. The systematic transposition of a series of points of a streamline enables us to find the corresponding flow about the wing profile, in which connection attention may be called to the fact that every point found is correlated with the corresponding point on the circle and therefore has a flow velocity comparable with that of the corresponding point of origin. If the velocity of the unsymmetrical cylindrical flow is thus known, the velocity for the corresponding profile streamline can be easily found. The above method was used for a number of streamlines, the results being given in Figures 3 and 4.

The equation for the flow velocity at any given point was previously developed. For the determination of the complex potential equation, we will start once more from the complex potential equation.

$$f(Z) = - V_\infty \left(Z + \frac{R_1^2}{Z}\right)$$

$$W_p = \frac{d\, f(Z)}{d\, Z} = - V_\infty \left(1 - \frac{R_1^2}{Z^2}\right) \tag{14}$$

If we put $Z = R\, e^{i\Lambda}$, that is, if we introduce polar coordinates with respect to the center of the circle, equation (14) is changed to

$$W_p = - V_\infty \left(1 - \frac{R_1^2}{R^2\, e^{i\Lambda}}\right)$$

or

$$W_p = - V_\infty\, (1 - m \cos 2\,\Lambda + i\, m \sin 2\,\Lambda) \tag{15}$$

if we put

$$\frac{R_1^2}{R^2} = m \tag{16}$$

If we put

$$c = 1 - m \tag{18}$$

the absolute value of W_p is calculated with

$$|W_p| = V_\infty \sqrt{(1 - m \cos 2\Lambda)^2 + m^2 \sin^2 2\Lambda}$$

$$|W_p| = V_\infty \sqrt{((1 - m) + 2 m \sin^2 \Lambda)^2 + 4 m^2 \sin^2 \Lambda \cos^2 \Lambda}$$

$$|W_p| = V_\infty \sqrt{c^2 + 4 c m \sin^2 \Lambda + 4 m^2 \sin^2 \Lambda} \tag{17}$$

In equation (16)

R_1 denotes the radius of the basic circle;

R denotes the length of the line from the center of the basic circle to the streamline point for which the velocity is to be calculated;

Λ denotes the angle between the line R and the parallel to the flow direction through the center of the basic circle.

If we measure the vertical distance h of the point on the streamline from the parallel to the direction of flow, we obtain

$$\frac{h}{R} = \sin \Lambda \tag{19}$$

Hence all the requisite numerical values can be determined from Figures 2 and 3, and W_p can be calculated by equation (17).

We obtain from equation (9) the circulatory velocity of a streamline point

$$\psi(Y,X) = -2 R_1 V_\infty \sin \alpha_{th} \ln \sqrt{X^2 + Y^2}$$

$$W_Z = \left|\frac{d}{dZ}\right| = -2 R_1 V_\infty \sin \alpha_{th} \frac{1}{\sqrt{X^2 + Y^2}}$$

$$|W_Z| = 2 \frac{R_1}{R} V_\infty \sin \alpha_{th} . \tag{20}$$

since

$$\sqrt{X^2 + Y^2} = R \tag{21}$$

These equations contain no unknown quantities, since $\sin \alpha_{th}$ was already known. The resultant velocity is the sum of the parallel-flow velocity and the circulatory velocity. It should be noted, however, that the latter can have both positive and negative values. This corresponds also to the purely logical conclusion, since the flow velocity on the lower side of the profile, due to opposite circulatory flow, is smaller than on the upper side where both flows have the same direction and strengthen each other.

The velocities $W(Z)$, thus calculated for the flow around the cylinder must be still further corrected for the wing profile. This is possible, due to the following considerations. As already mentioned, the flow variations in the profile plane z, for the short intervals of time, are proportional to the variations in the flow in the plane Z of the basic circle, when the variations in the Z values with respect to the z values are simultaneously considered.

$$\frac{d\chi}{dz} = \frac{d\overline{f}}{dZ}\,\frac{dZ}{dz} \tag{25}$$

or

$$|w(z)| = \left|\frac{W(Z)}{\frac{dz}{dZ}}\right| = \left|\frac{W(Z)}{1 - \frac{l^2}{Z^2}}\right| \tag{26}$$

because

$$z = Z - \frac{l^2}{Z} \quad \text{(functional equation of the Joukowsky function for conformal transformation)} \tag{27}$$

It then follows that

$$\frac{dz}{dZ} = 1 - \frac{l^2}{Z^2} .$$

The divisor of equation (6) can be written

$$\left|1 - \frac{l^2}{Z^2}\right| = \left|\frac{Z - \frac{l^2}{Z}}{Z}\right|$$

In the profile construction (Fig. 1) the line from the zero point to the outlying point on the streamline of the basic circle equals Z, and the mirrored line to the point in the inner circle equals l^2/Z. Therefore, the line connecting the points is the vectorial difference $Z - \frac{l^2}{Z}$. The quotient of the length of the line Z divided by this difference is therefore the correction factor by which the velocity on the circular profile must be multiplied, in order to obtain the correct velocity for the wing profile.

The values of W and of the correction factor K for the wing profile are plotted separately in Figure 5, in order to show their characteristic courses in terms of their local rela-

tion to the circular profile. This representation shows very clearly the strong influence of the profile on the region near its edge and the general balancing of the velocity differences with increasing distance from the profile edge. In its fundamental shape the curve of the correction factor forms the counterpart to the course of the velocity curves. The asymmetry of the profile shape is noticeable, however, this being the cause of the velocity variations as compared with the circular profile.

TABLE III

$$|W| = |W_p| + |W_z| \qquad \frac{|w|}{V_\infty} = \frac{|W|\ K}{V_\infty}$$

Streamline I

Point No.	1	2	3	4	5	6	7
W_p/V_∞	0.991	1.047	1.13	1.237	1.13	1.047	0.991
W_z/V_∞	0.119	0.132	0.143	0.154	0.143	0.132	0.119
W/V_∞	1.110	1.179	1.273	1.391	1.273	1.179	1.110
K	1.02	0.99	0.94	0.865	0.89	0.94	0.98
w/V_∞	1.12	1.17	1.2	1.202	1.13	1.11	1.09

Streamline II

W_p/V_∞	0.88	0.954	1.144	1.418	1.144	0.954	0.88
W_z/V_∞	0.143	0.167	0.189	0.206	0.189	0.167	0.143
W/V_∞	1.023	1.121	1.333	1.624	1.333	1.121	1.023
K	1.1	1.05	0.97	0.79	0.87	0.97	1.06
w/V_∞	1.125	1.179	1.297	1.284	1.156	1.109	1.086

TABLE III (cont'd)

$$|W| = |W_p| + |W_z| \qquad \frac{|w|}{V_\infty} = \frac{|W|\ K}{V_\infty}$$

Streamline III

Point No.	1	2	3	4	5	6	7
W_p/V_∞	0.75	0.592	0.912	1.826	0.608	0.592	0.75
W_z/V_∞	0.158	0.206	0.269	0.288	0.269	0.206	0.158
W/V_∞	0.908	0.798	1.181	2.114	1.181	0.798	0.908
K	1.2	1.36	1.16	0.67	0.89	1.39	1.26
w/V_∞	1.088	1.089	1.377	1.413	1.053	1.11	1.14

Streamline IV

W_p/V_∞	0.942	1.002	1.15	1.396	1.15	1.002	0.942
W_z/V_∞	0.132	0.156	0.181	0.199	0.181	0.156	0.132
W/V_∞	0.810	0.846	0.969	1.197	0.969	0.846	0.81
K	1.05	1.01	0.91	0.75	0.90	1.03	1.08
w/V_∞	0.85	0.857	0.897	0.898	0.876	0.875	0.872

Streamline V

W_p/V_∞	0.791	0.68	0.88	1.84	0.88	0.68	0.791
W_z/V_∞	0.154	0.202	0.27	0.291	0.27	0.202	0.154
W/V_∞	0.637	0.478	0.61	1.549	0.61	0.478	0.637
K	1.16	1.29	1.26	0.56	1.39	1.52	1.24
w/V_∞	0.74	0.619	0.766	0.875	0.846	0.726	0.79

Table III contains the velocities calculated for the five streamlines shown in Figure 5. The results are graphically represented in Figure 6. The values have already been introduced with the necessary distortion for the investigated wing profile at the corresponding locations, on the wing-profile streamlines, of the points coordinated with the circular streamlines. If cross sections are now made perpendicularly to the wing-profile

chord, it is then possible to plot the velocity drop for each section from the wing profile outward. The velocity on the contour of the wing profile itself can be easily calculated, since $R = R_1$ for this limiting case, so that equation (17) takes the form

$$|W_p| = 2 V_\infty \sin \Lambda \tag{22}$$

and equation (20)

$$|W_z| = 2 V_\infty \sin \alpha_{th} \tag{23}$$

or

$$|W| = 2 V_\infty (\sin \Lambda + \sin \alpha_{th}) \tag{24}$$

For this limiting case Trefftz has developed a simplified method of calculation. (Reference 1, page 18).

As the abscissa of the last-mentioned diagram (Fig. 7), we chose the percentile distance of each streamline point from the profile edge, referred to the maximum profile thickness as unity and, as the ordinate, the velocity corresponding to each point. In this way we obtained, for any dimensions of the profile, a practical representation, which can also be used for determining the velocity distribution for any propeller arrangement in the plane of any profile section.

Analysis of the Results

On comparing the increase in the velocities on the streamlines I, II and III and the velocity drop of the streamlines from within outward, the regularity of the velocity distribution is obvious. The effect of the profile disturbance on the flow

increases from within outward, attaining its maximum value at the profile edge. The velocity increment in the direction of a streamline is distributed in such a way that the velocity of a streamline attains its highest value in the forward third of the profile and decreases from that point toward the trailing edge. The theoretically calculable maximum velocity is about 1.6 times the flow velocity.

The effect of the profile on the flow is noticeable for some distance fore and aft of the profile. The velocity on the edge of the profile drops to zero at the points of full dynamic pressure. Its effect can still be observed in the velocity of the streamlines near the profile edge, since there is a considerable velocity drop in these streamlines fore and aft of the profile. Consequently there is, in the streamlines fore and aft of this point a further velocity increase, before this velocity drops, at the corresponding distance, to its normal value of V_∞. In more remote streamlines a certain equalization has already taken place, so that a constant velocity drop is observable with increasing distance from the profile. Like phenomena are observable in streamlines IV and V on the lower side of the profile.

If the individual planes of the profile sections are examined in the light of these results, there is to be expected, for propellers mounted above the profile, a relatively uniform velocity distribution in the intersecting plane a (Fig. 7) in front of the profile. The plane b at the height of the leading edge of the profile is also favorable for propellers, provid-

ed the blade tips do not come too close to the profile edge. In plane c, on the contrary, great velocity differences and corresponding unequal deflections of the propeller-disk surfaces are to be expected. Conditions similar to those for c obtain for planes d to g, but the conditions gradually improve toward the trailing edge of the wing. Near the profile edge the plane h shows a certain velocity drop, which probably has no effect, however, at the customary minimum distance. The trailing edge of the profile offers very favorable velocity relations. The plane k shows, at about the height of the trailing edge, a small velocity increase for this portion of the field, the efficiency of which depends on the height of the propeller.

The conditions are similar on both sides of the profile. The only exceptions are the cross sections a and i, whose velocity fields are subject to certain irregularities. The planes c, g, and h likewise exhibit certain velocity differences, but they are not nearly so great as on the upper side.

The calculated order of magnitude of the velocity variations must naturally be adopted with caution and should first be verified by practical experiments, since practical conditions often differ from the ideal theoretical conditions and produce effects which cannot be taken care of mathematically. It might be concluded from the results of the Göttingen pressure measurements on wing profiles that the tendency of the curve is correct, since the distribution of the pressure along the profile chord was

there found to be similar to that calculated here for the velocity distribution, and since the pressure distribution, as kinetic energy, is a function of the velocity.

A knowledge of the profile effect on the flow velocity is also of some importance for the application of dynamic-pressure meters, because it enables one to estimate the minimum distance from the wing for measuring the least possible affected flow velocity.

References

1. Trefftz, E. : Z.F.M., 1913, p. 130 (For translation, see N.A.C.A. Technical Memorandum No. 336; page 17: "Graphic Construction of Joukowski Wings," 1925.
2. Schrenk, O. : Z.F.M., 1927, p. 276 (For translation, see McCook Field Memo. Report No. 215: "Elementary Discussion of Joukowski's Theory of Profiles," 1927.
3. Schrenk, O. : Z.F.M., 1927, p. 225: (For translation, see N.A.C.A. Technical Memorandum No. 422: "Systematic Investigation of Joukowski Wing Sections," 1927.
4. Eberhard, : "Einführung in die theoretische Aerodynamik."

Translation by
National Advisory Committee
for Aeronautics.

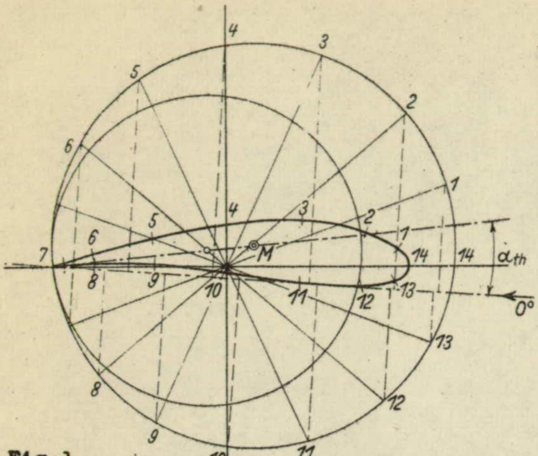

Fig.1

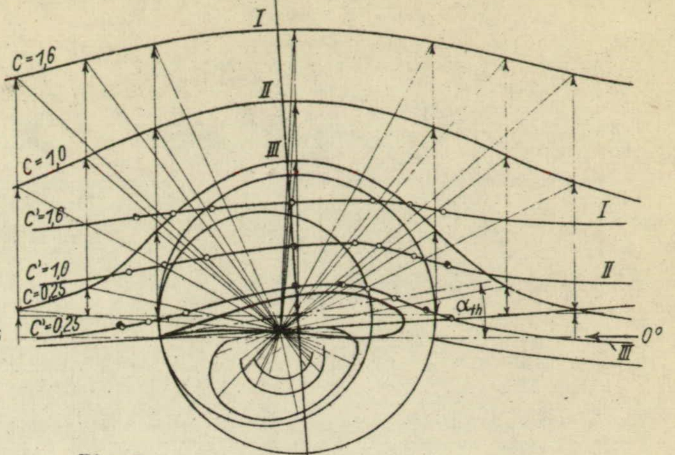

Fig.3

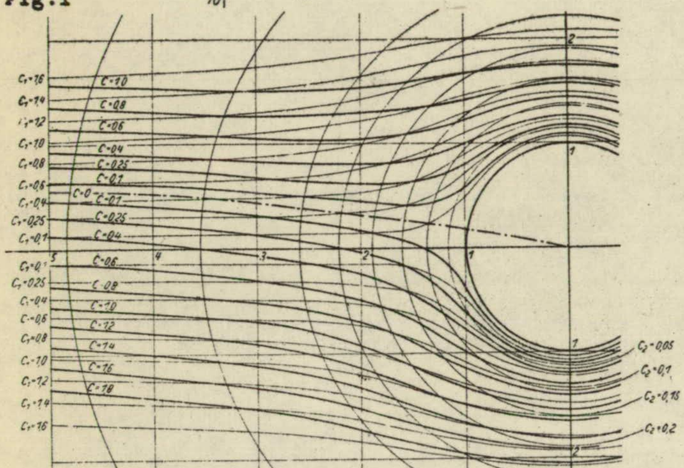

Fig.2

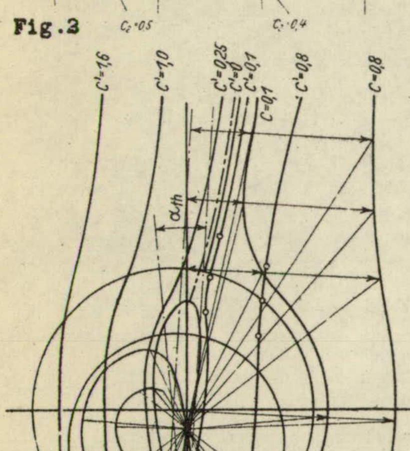

Fig.4

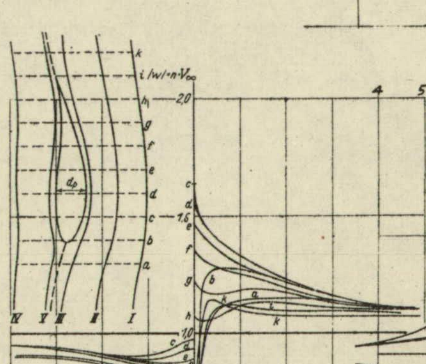

Fig.7

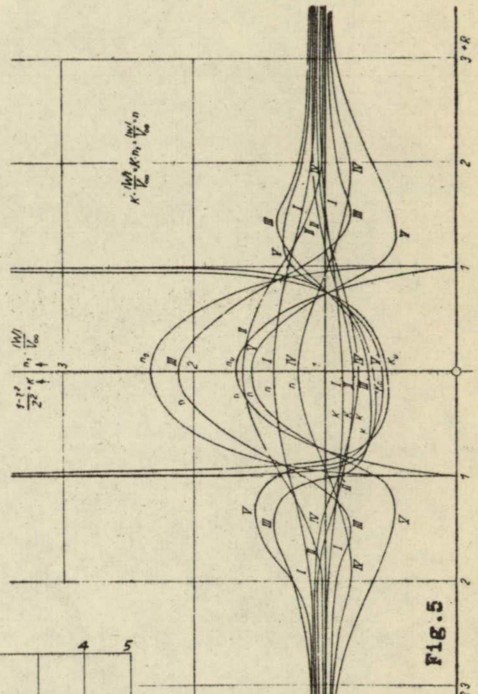

Fig.5

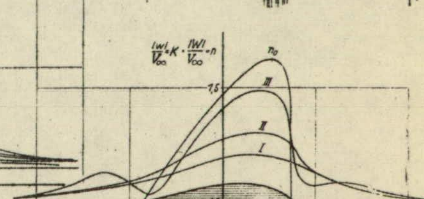

Fig.6